Journal of Applied Mechanics and Technical Physics, Vol. 35, No. 2, 1994

SOME EXACT SOLUTIONS OF THE DISPLACEMENT PROBLEM WHEN FILTERING A GAS-CONDENSATE MIXTURE

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UDC 532.546

When developing gas-condensate deposits, a mode of operation in which the layer pressure is maintained is widely used at the present time. Gas is pumped into the layer in order to prevent the precipitation of hydrocarbons in the liquid phase due to retrograde condensation [1, 2]. The pressure in the layered mixture is maintained close to (above or slightly below) the pressure at the beginning of condensation. To calculate the features of this development it is desirable to have a solution of the corresponding model problems of displacement. These problems have been considered into a number of publications (see the review in [3]), but the lack of adequate accurate results on the problem of displacement with phase transitions is still being felt.

In this paper we obtain exact solutions (in quadratures) of the one-dimensional problem of multicomponent two-phase filtration with phase transitions. Particular attention is given to the process of the displacement of a two-phase gas-liquid mixture by a gas.

Consider the one-dimensional steady filtration of a multicomponent two-phase mixture with phase transitions in a uniform isotropic porous medium. The following laws of conservation must be satisfied.

$$\frac{\partial}{\partial t}[m((1 - s)n_{ig} + sn_{il})] + \frac{\partial}{\partial x}J_i = 0,\ i = 1,\dots,N, \tag{1}$$

where t is the time, x is the Cartesian coordinate, N is the number of components (N > 1), m is the porosity, s is the saturation of the porous space by the liquid phase, n_{ig} is the density of molecules of the i-th component in the gaseous phase, n_{il} is the density of molecules of the i-th component in the liquid phase, and J_i is the flow of molecules of the i-th component. We will use the following expression for J_i:

$$J_i = -k(n_{ig} f_g \mu_g^{-1} + n_{il} f_l \mu_l^{-1}) \frac{\partial p}{\partial x}. \tag{2}$$

Here k is the permeability, f_g is the phase permeability of the gaseous phase, f_l is the phase permeability of the liquid phase, μ_g is the viscosity of the gas, μ_l is the viscosity of the liquid, and p is the pressure. We will neglect capillary forces.

The dynamic equations (1) and (2) must be supplemented by the conditions of phase equilibrium

$$\varkappa_{ig} = \varkappa_{il},\ i = 1,\dots,N. \tag{3}$$

Problem (1)-(3) becomes closed if we specify the equation of state

$$p = p(T,n_i); \tag{4}$$

and the expressions for the chemical potentials

$$\varkappa_i = \varkappa_i(T,n_j); \tag{5}$$

the viscosities

$$\mu_g = \mu_g(T,n_{ig}),\ \mu_l = \mu_l(T,n_{il}); \tag{6}$$

Moscow. Translated from Prikladnaya Mekhanika i Tekhnicheskaya Fizika, No. 2, pp. 82-85, March-April, 1994. Original article submitted May 7, 1993.

and the phase permeabilities

$$f_g = f_g(s),\ f_l = f_l(s).$$

In formulas (4)-(6) T is the temperature, which will be assumed to be constant. Note also that in expression (2) $p = p(T, n_{ig}) = p(T, n_{il})$, while in Eq. (3) $\varkappa_{ig} = \varkappa_i(T, n_{jg})$, $\varkappa_{il} = \varkappa_i(T, n_{jl})$.

We will seek solutions which depend only on the parameter $\xi = x - Vt$, $V = \text{const}$. We then obtain from Eqs. (1) and (2)

$$\frac{d}{d\xi}\left(n_{ig}(kf_g\mu_g^{-1}\frac{dp}{d\xi} + m(1 - s)V) + n_{il}(kf_l\mu_l^{-1}\frac{dp}{d\xi} + msV)\right) = 0,\ i = 1,\dots,N. \tag{7}$$

We will introduce the following notation: $n_g = \sum_{i=1}^{N} n_{ig}$ is the density of the molecules in the gaseous phase, $n_l = \sum_{i=1}^{N} n_{ij}$ is the density of the molecules in the liquid phase, $c_{ig} = n_{ig}/n_g$ is the molar fraction of the molecules in the gaseous phase, and $c_{ij} = n_{il}/n_l$ is molar fraction of the molecules in the liquid phase. As can easily be seen, Eqs. (7) have the following first integrals:

$$\begin{gathered} Gc_{ig} + Lc_{il} = q_i,\ i = 1,\dots,N, \\ G = n_g(kf_g\mu_g^{-1}\frac{dp}{d\xi} + m(1 - s)V), \\ L = n_l(kf_l\mu_l^{-1}\frac{dp}{d\xi} + msV). \end{gathered} \tag{8}$$

We will put $q = \sum_{i=1}^{N} q_i$. The quantity $(-q)$ has the physical meaning of the total flow of molecules in a moving frame of reference. The following equation follows from Eqs. (8):

$$G + L = q. \tag{9}$$

We will first consider the case when $q = 0$, but the quantities q_i exist and are nonzero. Then $G = -L$ and Eqs. (8) becomes

$$G(c_{ig} - c_{il}) = q_i,\ i = 1,\dots,N. \tag{10}$$

Since we are interested in the problem of filtration, we will take into account the purely thermodynamic problem of phase equilibrium for a multicomponent mixture, solved on the basis of the experiment or by calculation using the semiempirical equations of state [2, 4, 5]. The system of equations (10) must then be interpreted as the problem of finding a two-phase state in which the vector $a_i = c_{ig} - c_{il}$, for a known connection between c_{ig} and c_{il}, is collinear with the specified vector q_i, This problem may: a) have no solution, b) have a unique solution, and c) have a nonunique solution. In the last two cases we can obtain the functions $G = \nu(p)$, $c_{ig} = c_{ig}(p)$, $c_{il} = c_{il}(p)$, which satisfy system (10). Then, from the equation $G = \nu$ and (9) we obtain the following ordinary differential equations for $p = p(\xi)$:

$$\frac{dp}{d\xi} = -mVk^{-1}((1 - s)n_g + sn_l)(f_gn_g\mu_g^{-1} + f_ln_l\mu_l^{-1}), \tag{11}$$

and also the equation for determining the saturation s as a function of p

$$(n_gf_g\mu_g^{-1} + n_lf_l\mu_l^{-1})\nu + n_gn_lmV(sf_g\mu_g^{-1} - (1 - s)f_l\mu_l^{-1}) = 0. \tag{12}$$

Substituting the solution of Eq. (12) into the right-hand side of Eq. (11) we obtain an autonomous equation of the form $dp/d\xi = F(p)$, which can be solved in a standard way in quadratures.

Suppose now that all the partial flows are equal to zero: $q_i = 0$. The $G = L = 0$ and we again obtain Eq. (11) with an equation for determining s:

$$\mu_g(1 - s)f_l - \mu_lsf_g = 0. \tag{13}$$

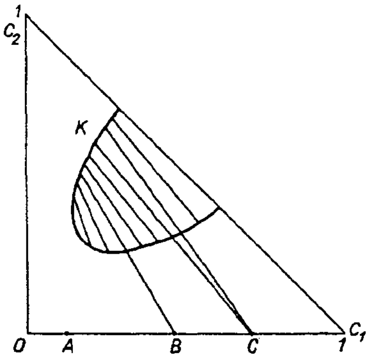

Fig. 1

Analysis shows that under practical conditions Eq. (13) cannot have nontrivial solutions. Thus, assuming $f_l = s$, $f_g = 1 - s$, $\mu_g = \text{const}$, $\mu_l = \text{const}$ ($\mu_l > \mu_g$), we seen that (13) is only satisfied when s = 0.1.

We will now consider the case when $q \neq 0$. We put $\eta_i = q_i/q$, $g = G/q$, $l = L/q$. It follows from system (8) that

$$gc_{ig} + lc_{il} = \eta_i,\ i = 1,\ldots,N. \tag{14}$$

The parameters g and l are related as follows:

$$g + l = 1. \tag{15}$$

Suppose we have a pair of compounds c_{ig} and c_{il}, connected by the conditions of phase equilibrium (3) for a specified pressure p. Then the left-hand side of Eq. (14), taking Eq. (15) into account, specifies a one-dimensional subspace in an N-dimensional affine space. The ordered set of numbers η_i may or may not belong to his subspace. In the first case the parameters g and l are defined uniquely.

Hence, the problem reduces to finding a two-phase state for a specified pressure p such that the pair of vectors ($c_{ig} - \eta_i$), ($c_{il} - \eta_i$) is collinear, i.e. it reduces to a purely thermodynamic problem. The following situations are possible: 1) there is no solution, 2) a solution exists and is unique, and 3) a solution exists but is not unique. In cases 2 and 3 certain relations $c_{ig} = c_{ig}(p)$, $c_{il} = c_{il}(p)$, $l = F(p)$, $g = 1 - F(p)$, exist which satisfy relations (3) and (14). Hence, we also obtain from Eq. (9) the following ordinary differential equations for the pressure:

$$\frac{dp}{d\xi} = k^{-1}(q - mV((1 - s)n_g + sn_l))(f_g n_g \mu_g^{-1} + f_l n_l \mu_l^{-1}), \tag{16}$$

and also an equation for determining the saturation

$$q(1 - F)n_l f_l \mu_l^{-1} - qFn_g f_g \mu_g^{-1} = n_g n_l mV((1 - s)f_l \mu_l^{-1} - sf_g \mu_g^{-1}). \tag{17}$$

We will discuss in more detail the case when the solution of Eqs. (10) (or (14)) is not unique. Then the solutions $c_{ig} = c_{ig}(p)$, $c_{il} = c_{il}(p)$, $G = \nu(p)$ (or $l = F(p)$) exist with discontinuities. The corresponding flow in the porous medium has jumps in concentration and saturation. Hence, the jumps are connected with the nonuniqueness of the solution of the purely thermodynamic problem and can be observed when analyzing the phase diagrams in concentration space. As an example consider a three-component system (see Fig. 1), in which there is a region of two-phase states bounded by the binodal K. Suppose the vector η_i corresponds to the vector of the mixture concentration, in which $c_2 = 0$. For point A there are no two-phase solutions of system (14), for point B there is a unique two-phase state, and for point C the two-phase state is not unique. For all three points there is an obvious single-phase solution with $g = 1$, $l = 0$. The cases of points B and C can then be interpreted as the displacement of a certain two-phase mixture by a single-phase mixture (a gas or a liquid). In fact, for points B and C we can construct a solution with a discontinuity at $\xi = 0$, for which $V > 0$, where for $\xi < 0$ single-phase flow is obtained, while for $\xi > 0$ two phase flow is obtained.

It can be seen from this example that in the general case, when at a certain pressure p_* system (14) possesses single-phase and two-phase solutions, we can construct a solution of the filtration problem with a jump at the pressure p_*. A transition from single-phase flow to two-phase flow occurs on the jump.

To find a specific analytic form of $p = p(\xi)$ it is necessary to solve Eqs. (16) and (17). If the phase permeabilities depend linearly on the saturation and we know how F, n_g, n_{ll}, μ_g, μ_l depend on p, the quadratic equation (17) in the saturation s can be solved explicitly and we can write the following formal implicit solution:

$$\xi = \int dp/\Phi(p),$$

where $\Phi = \Phi(p)$ is the explicit expression for the right-hand side of Eq. (16). When f_l and f_g depend nonlinearly on s, problem (16), (17) can be solved numerically.

The solution obtained can be used to describe the processes that occur in gas-condensate and gas-petroleum deposits when they are being exploited.

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